

**Department:**

Mathematics

**Course Description:**

This course is the final course in the three-semester sequence of calculus courses. This course is designed to prepare students to be successful in Differential Equations, Vector Analysis, Statics, Dynamics, and other upper-level mathematics, science, and engineering courses. The course consists of a thorough study of polar coordinates and parametric equations, vector analysis in calculus problems, vector-valued functions, partial derivatives, centroids, directional derivatives, gradients, and multiple integrals including double integrals, triple integrals, changing variables involving polar coordinates, center of mass and moments of inertia, and many applications. In addition, there will be a thorough study of multiple integrals and their applications, including in cylindrical and spherical coordinates and change of variables using Jacobians. Topics from the field of vector analysis, such as vector fields, line integrals, Green's Theorem applications, surface integrals including applications and flux, and the use of matrices in various operations will also be covered.

**Course Competencies:**

Upon completion of the course, the student should be able to:

1. Explain polar coordinates, analyze polar graphs, and find area enclosed by regions described by polar coordinates.
2. Find dot products of two vectors, including projections; find the cross product of two vectors in space, including calculation of the scalar triple product; and use a parametric equation to find tangent lines and arc length.
3. Use vectors to derive equations of planes in 3-space, including the use of these equations to solve geometric problems.
4. Find limits, derivatives, and integrals for vector-valued functions.
5. Find unit tangent, normal, and binormal vectors, including related applications, especially curvature.
6. Find limits and establish continuity for functions of two or more variables.
7. Find partial derivatives and differentials for functions of two or more variables.
8. Derive and use versions of the chain rule for functions of two or more variables to find partial derivatives.
9. Use directional derivatives and gradients in functions of two or three variables for applications, including using the chain rule, to assist with finding tangent planes, normal lines, and extrema of functions of two variables, including applications of extrema of functions of two or more variables.
10. Use Lagrange multipliers to maximize or minimize a function subjected to certain constraints.
11. Use double integrals over rectangular, nonrectangular, and polar coordinates to solve problems.
12. Use integration techniques to find the centroid of a given region, the mass and the center of gravity of the lamina, the centroid of a solid, and surface area.

13. Use triple integrals in cylindrical and spherical coordinates to solve problems.
14. Change variables in multiple integrals to find the Jacobian.
15. Use a specific transformation technique to find a double integral over a triangular region with specific set of vertices.
16. Sketch and use vector field techniques to perform various operations, such as divergence and curl.
17. Perform line integrals.
18. Explain Green's Theorem and apply it in finding areas.
19. Perform surface integrals and apply surface integrals to applications such as flux.
20. Explain the Divergence Theorem and use it in finding flux across a surface.
21. Explain Stokes' Theorem and its relation to Green's Theorem.

## Course Content:

- A. Parametric Equations and Polar Coordinates
  1. Introduction to Polar Coordinates
  2. Parametrizations of Plane Curves
  3. Calculus with Parametric Curves
  4. Graphing in Polar Coordinates
  5. Areas in Polar Coordinates
  6. Lengths in Polar Coordinates
  7. Conic Sections
    - a. Parabolas
    - b. Ellipses
    - c. Hyperbolas
    - d. Applications of each conic section
  8. Conics in Polar Coordinates
- B. Vectors and the Geometry of Space
  1. Three-Dimensional Coordinate Systems
  2. Vectors
    - a. Definitions
    - b. Vector Algebra Operations
    - c. Applications of vectors
  3. The Dot Product
  4. Vector Projections
  5. The Cross Product
  6. Lines and Planes in Space
  7. Cylinders and Quadric Surfaces
- C. Vector-Valued Functions and Motion in Space
  1. Curves in Space and Their Tangents
  2. Integrals of Vector Functions
  3. Projectile Motion
  4. Arc Length in Space
  5. Curvature and Normal Vectors of a Curve
  6. Components of Acceleration
    - a. Tangential
    - b. Normal
  7. Polar Coordinates and Physics
    - a. Velocity
    - b. Acceleration
    - c. Kepler's Laws 1 and 2
- D. Partial Derivatives
  1. Functions of Several Variables

- a. Graphs
  - b. Level curves and surfaces
  - c. Contours
  - d. Computer graphing
- 2. Limits and Continuity in Higher Dimensions
- 3. Partial Derivatives and Continuity
  - a. First-order
  - b. Second-order
  - c. Higher-order
- 4. The Chain Rule
  - a. For two Independent variables
  - b. For three Independent variables
- 5. Directional Derivatives and Gradient Vectors
  - a. Directional derivatives in the plane
  - b. Calculation and the gradient vector
  - c. Gradients and tangents to level curves
- 6. Tangent Planes and Differentials
  - a. Tangent planes and differentials
  - b. The normal line
  - c. The plane tangent to a surface
  - d. Linearization of a function
  - e. The total differential
- 7. Extreme Values and Saddle Points
  - a. Derivative tests for local extreme values
  - b. Critical points and saddle points
  - c. The second derivative test for local extreme values
  - d. Absolute maxima and minima
- 8. Lagrange Multipliers
  - a. Constrained maxima and minima
  - b. The Method of Lagrange Multipliers
- 9. Taylor's Formula for Two Variables
- 10. Partial Derivatives with Constrained Variables
- E. Multiple Integrals
  - 1. Double and Iterated Integrals over Rectangles
    - a. Double integrals
    - b. Fubini's Theorem for Double Integrals
  - 2. Double Integrals over General Regions
    - a. Double integrals over bounded, nonrectangular regions
    - b. Stronger form of Fubini's Theorem
    - c. Finding limits for Integrating (with cross-sections)
    - d. Properties of double integrals
  - 3. Area by Double Integration
    - a. Areas of bounded regions in the plane
    - b. Average value
  - 4. Double Integrals in Polar Form
    - a. Integrals in Polar Coordinates
    - b. Finding limits of integration
    - c. Area in polar coordinates
    - d. Changing Cartesian integrals into polar integrals
  - 5. Triple Integrals in Rectangular Coordinates
    - a. Finding triple integrals
    - b. Volumes of a region in space

- c. Finding limits of integration in three-space order
    - d. Average value of a function in space
  - 6. Moments and Centers of Mass
    - a. Masses and First Moments
    - b. Three-dimensional solid and two-dimensional plate
    - c. Moments of inertia
  - 7. Triple Integrals in Cylindrical and Spherical Coordinates
    - a. Integration in cylindrical coordinates
    - b. Spherical coordinates and integration
  - 8. Substitutions in Multiple Integrals
    - a. Double integrals
    - b. Jacobian determinants
    - c. Triple integrals
- F. Integration in Vector Fields
  - 1. Line Integrals
    - a. Evaluating a line integral
    - b. Mass and moment calculations
    - c. Line integrals in the plane
  - 2. Vector Fields and Line Integrals: Work, Circulation, and Flux
    - a. Vector fields
    - b. Gradient fields
    - c. Line integrals in vector fields
    - d. Work done by a force over a curve in space
    - e. Flow integrals
  - 3. Path Independence (time permitting)
  - 4. Green's Theorem in the Plane
    - a. Divergence
    - b. Circulation density
  - 5. Surfaces and Area
  - 6. Surface Integrals
  - 7. Stokes' Theorem

## Learning Assessments:

Course competencies will be assessed by written examinations covering all course material, including regular hour-long exams and a required, comprehensive final exam. Additionally, assessment may also occur through any of the following at the discretion of the instructor: regular collection of homework, in-class work, quizzes, and various projects.

## Instructional Materials:

Textbook: Briggs, W., Cochran, L., Gillett, B., & Schultz, E. (2015). *Calculus: Early Transcendentals* (2<sup>nd</sup> ed.). Boston, MA: Pearson Education. ISBN-13: 978-0321947345.

### Guidelines for Requesting Accommodations Based on Documented Disability or Medical Condition

It is the intention of Highland Community College to work toward full compliance with the Americans with Disabilities Act, to make instructional programs accessible to all people, and to provide reasonable accommodations according to the law.

Students should understand that it is their responsibility to self-identify their need(s) for accommodation and that they must provide current, comprehensive diagnosis of a specific disability or medical condition from a qualified professional in order to receive services. Documentation must include specific recommendations for accommodation(s). Documentation should be

provided in a timely manner prior to or early in the semester so that the requested accommodation can be considered and, if warranted, arranged.

In order to begin the process all students **must** complete the “Disabilities Self-Identification Form” on our [Disability Services website](#).

This form can also be accessed at the Highland Community College homepage under Students Services/Student Resources/Disability Service or by contacting the Disabilities Coordinator.

#### **A Note on Harassment, Discrimination and Sexual Misconduct**

Highland Community College seeks to assure all community members learn and work in a welcoming and inclusive environment. Title VII, Title IX, and College policy prohibit harassment, discrimination and sexual misconduct. Highland Community College encourages anyone experiencing harassment, discrimination or sexual misconduct to talk to report to the Vice President for Student Services, the Human Resources Director or complete an [online report](#) about what happened so that they can get the support they need and Highland Community College can respond appropriately.

There are both confidential and non-confidential resources and reporting options available to you. Highland Community College is legally obligated to respond to reports of sexual misconduct, and therefore we cannot guarantee the confidentiality of a report, unless made to a confidential resource. Responses may vary from support services to formal investigations. As a faculty member, I am required to report incidents of sexual misconduct and thus cannot guarantee confidentiality. I must provide our Title IX coordinator with relevant details such as the names of those involved in the incident. For more information about policies and resources or reporting options, please review our [Equity Grievance Policy](#).